Calculus III Formula Sheet – Briggs, Cochran – Updated Fall 2016

$$W = |\mathbf{F}|\,|\mathbf{d}| \cos\theta = \mathbf{F}\cdot\mathbf{d}$$

$$|\tau| = |\mathbf{r}\times\mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta,$$

$$L = \int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\; d\theta$$

$$\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$$

$$\mathbf{B} = \mathbf{T}\times\mathbf{N} = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi \qquad \rho^2 = x^2 + y^2 + z^2$$

$$m = \iint_R \rho(x,\,y)\,dA$$

$$\bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_R y\,\rho(x,\,y)\,dA$$

$$\bar{x} = \frac{M_{yz}}{m} = \frac{1}{m}\iiint_D x\,\rho(x,\,y,\,z)\,dV,$$

$$\bar{y} = \frac{M_{xz}}{m} = \frac{1}{m}\iiint_D y\,\rho(x,\,y,\,z)\,dV$$

$$\bar{z} = \frac{M_{xy}}{m} = \frac{1}{m}\iiint_D z\,\rho(x,\,y,\,z)\,dV$$

$$J(u,\,v) = \frac{\partial(x,\,y)}{\partial(u,\,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}.$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_D \nabla\cdot\mathbf{F}\,dV$$

$$\oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C (x\,dy - y\,dx)$$

$$\iint_S f(x,\,y,\,z)\,dS = \iint_R f(x(u,\,v),\,y(u,\,v),\,z(u,\,v))\,|\mathbf{t}_u\times\mathbf{t}_v|\,dA.$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_R \mathbf{F}\cdot(\mathbf{t}_u\times\mathbf{t}_v)\,dA$$

$$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta$$

$$a_N = \kappa\,|\mathbf{v}|^2 = \frac{|\mathbf{a}\times\mathbf{v}|}{|\mathbf{v}|} \text{ and } a_T = \frac{d^2s}{dt^2} = \frac{\mathbf{a}\cdot\mathbf{v}}{|\mathbf{v}|}$$

$$\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$$

$$\kappa = \frac{|\mathbf{a}\times\mathbf{v}|}{|\mathbf{v}|^3}$$

$$\tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N} = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{a}'}{|\mathbf{v}\times\mathbf{a}|^2} = \frac{(\mathbf{r}'\times\mathbf{r}'')\cdot\mathbf{r}'''}{|\mathbf{r}'\times\mathbf{r}''|^2}$$

$$\iiint_D f(\rho,\,\phi,\,\theta)\,dV = \int_\alpha^\beta \int_a^b \int_{g(\phi,\theta)}^{h(\phi,\theta)} f(\rho,\,\phi,\,\theta)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_R x\,\rho(x,\,y)\,dA$$

$$L(x,\,y) = f_x(a,\,b)\,(x-a) + f_y(a,\,b)\,(y-b) + f(a,\,b)$$

$$\begin{aligned}\int_a^b \mathbf{F}\cdot\mathbf{r}'(t)\,dt &= \int_a^b (f\,x'(t) + g\,y'(t) + h\,z'(t))\,dt \\ &= \int_C f\,dx + g\,dy + h\,dz \\ &= \int_C \mathbf{F}\cdot d\mathbf{r}\end{aligned}$$

$$\int_C \mathbf{F}\cdot\mathbf{n}\,ds = \int_a^b (f\,y'(t) - g\,x'(t))\,dt$$

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA.$$

$$\oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \oint_C f\,dy - g\,dx = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA$$

$$\iint_S f(x,\,y,\,z)\,dS = \iint_R f(x,\,y,\,g(x,\,y))\sqrt{{z_x}^2 + {z_y}^2 + 1}\;\,dA.$$

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$$